We will start by playing a game that gave rise to one of the main behavioral model, namely the quantal response model (QRE).

As usual, the challenge is to have the following features interacting with each other:

- Your feeling as a subject
- Data interpretation
- Link to theory
- Econometric aspects
Let’s play!: the centipede game

- It is a two player game
- Play the first version of the game with one partner
- Then play the other version (6-steps) with someone else
- Please note the outcomes of the games so that we can analyze results together
Let’s play!: the centipede game

*Figure 10.* The four-move centipede game (McKelvey-Palfrey, 1992).

*Figure 11.* The six-move centipede game (McKelvey-Palfrey, 1992).
Centipede: experimental results

![Bar chart showing take probabilities for a 4-move game. The bars represent P1, P2, P3, and P4 with corresponding probabilities: P1 = 0.1, P2 = 0.4, P3 = 0.6, P4 = 0.8.](image)
Analysis

- This game creates a trade-off between
- Pass to reach higher gains (but with a chance to loose)
- Take to secure a smaller gain
- Players may want to take some risks or not. Note that risk here refers to strategic uncertainty
- The idea is to give this intuitive notion a more precise meaning
- This is the purpose of the QRE model
The (logit)QRE

- The idea is that players are making mistakes when trying to best-respond to their subjective beliefs.
- It is assumed that errors are less likely when their cost is high.
- It is common knowledge among players that mistakes will occur.
- So a player will best-respond to other players making mistakes, while anticipating he will make mistakes himself.
My favorite example :) 

- There was a huge debate in France regarding same-sex marriage
- The left wing gouverment was in favour of it
- While part of the right wing was strongly opposed to it
- One strong opponent was Henri Guaino

Guaino and three others ended up voting in favor of the law!!!
QRE as a fixed point

- Individuals are more likely to select better choices than worse choices, but do not necessarily succeed in selecting the very best choice.
- Formally, a quantal response function maps the vector of expected payoffs from available choices into a vector of choice probabilities that is monotone in the expected payoffs.
- A quantal response equilibrium (QRE) imposes the requirement that the beliefs match the equilibrium choice probabilities.
- Thus, QRE requires solving for a fixed point in the choice probabilities, analogous to the Nash equilibrium.
A quick refresher course in probabilistic choice models

- If utility functions are fully observable and deterministic, each time we observe that an individual chooses A over B, we can infer that $U(A) > U(B)$
- But, if there is some uncertainty regarding the exact shape of utility functions because:
  - Players may have incomplete or incorrect information or misperceptions about the attributes of the alternatives (e.g. they can make pure mistakes like Guaino, have some misunderstanding regarding the rules of the game, etc)
  - The observer has an inadequate understanding of the function the individual uses to evaluate the utility of each alternative (e.g. players may be altruistic)
A quick refresher course in probabilistic choice models

- Models that take account of this lack of information on the part of the analyst are called random utility or probabilistic choice models.
- They decompose the utility function into an observable part, \( \beta x_A \) and an unobservable one \( \epsilon_A \):

\[
u_A = \beta x_A + \epsilon_A
\]

- So, the probability of choosing \( A \) over \( B \) is

\[
Pr(u_A > u_B) = Pr(\beta x_A - \beta x_B > \epsilon_B - \epsilon_A)
\]

- In most applications in game, utility is simply measured by the monetary amounts, since no particular scale is available for utility and one has to be chosen
The logistic approach

If the difference in unobservable, $\epsilon_B - \epsilon_A$, is assumed to follow a logistic distribution (e.g. $\epsilon_B$ and $\epsilon_A$ follow a Gumbel distribution) the probability of choosing option A over B is given by the logistic function as a function of the difference in utility.

If both options have the same value, the decision maker is indifferent between the two alternatives, and the larger the difference the more deterministic the choice.
The logistic approach

- The nice thing with the logistic approach is that we get a closed form for probability of choosing alternative $i$ among a set of $J$ alternatives:

$$p_i = \frac{e^{\lambda u_i}}{\sum_{k=1}^{J} e^{\lambda u_k}}$$

- The key parameter is $\lambda$: it describes the ”steepness” of the logistic function.
An extension to normal form games: the logistic-QRE

- Now let us assume that Alice and Bob are playing a normal form game.
- Alice knows that Bob will use a logistic function to choose. Bob also knows that Alice will do the same, and so on...
- We also assume that players know the correct value of $\lambda$
- They would then solve for a sort of noisy-Nash, but with a specific structure of the noise (e.g. logistic)
A simple example

- Let’s first consider a simple 2x2 game that will help us understand the logic of QRE.
- There are two players, row and column.
- We can first compute mixed strategy in this game.

<table>
<thead>
<tr>
<th></th>
<th>Left</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>4,1</td>
<td>1,2</td>
</tr>
<tr>
<td>Bottom</td>
<td>1,2</td>
<td>2,1</td>
</tr>
</tbody>
</table>
Row will choose $P_T$ that maximizes
$$4P_T P_L + 1P_T (1 - P_L) + 1(1 - P_T) P_L + 2(1 - P_T)(1 - P_L)$$
i.e. $P_T[4P_L - 1] + Cste$

The equilibrium condition is $P_L = 1/4$

So Row will choose $P_T = 1$ if $Eu(T) > Eu(B)$ and so on...

Column will choose $P_L$ that maximizes
$$1P_T P_L + 2P_T (1 - P_L) + 2(1 - P_T) P_L + 1(1 - P_T)(1 - P_L)$$
i.e. $P_L[1 - 2P_T] + Cste$
A simple example

- We find the best-response correspondance
Logit best-response

Now let us assume that instead of deterministic strategies, players use logit best-response.

What equation would you need to solve to get these curves?
Logit Best-response

We need to solve the following system of equation for a fixed $\lambda$

- In the logistic case we get a simple system of equation to solve:

$$p_T(\lambda) = \frac{e^\lambda[(4p_L+1(1-p_L)]}{e^\lambda[(4p_L+1(1-p_L)] + e^\lambda[1(1-p_L)]$$

$$p_R(\lambda) = \frac{e^\lambda[1p_T+2(1-p_T)]}{e^\lambda[1p_T+2(1-p_T)] + e^\lambda[2p_T+1(1-p_T)]}$$

- So the best response is not deterministic, there is always a chance that Row will play Top
A simple example

- We thus get a different equilibrium for each value of $\lambda$
A simple example

- We can thus plot the equilibrium path when $\lambda$ varies

So, once we get an observed value for $p_T$ and $p_L$ we can find the value of $\lambda$ that fits best according to some criteria of good fit.
Testing the model: how does $\lambda$ evolves when players are learning?

- The following game is played 200 times
- It is a zero sum game
- The following matrix specifies payments made by player 2 to player 1

$$
\begin{array}{c|ccc}
 & B_1 & B_2 & B_3 \\
\hline
A_1 & 15 & 0 & -2 \\
A_2 & 0 & -15 & -1 \\
A_3 & 1 & 2 & 0 \\
\end{array}
$$

- Does this game have a Nash Equilibrium?
Testing the model: how does $\lambda$ evolves when players are learning?

- Theoretical analysis allow us to plot the probability of each strategy according to $\lambda$
- The average, across players and across 10 periods, is represented by a black dot
The notion of Logit-QRE can be extended to sequential games, like the centipede.

The following table provides some estimates for various models based on the previous data:

- NNM is a model in which players are assumed to play Nash equilibrium with some strategy and to play randomly otherwise (using an uniform distribution).
- Zauner model, similar but using a normally distributed noise (instead of weibull/logit).
- The two-parameter model is an extended version of QRE including altruism.
Back to centipede

- Models with the same number of parameters can be compared on the basis of log-likelihood

<table>
<thead>
<tr>
<th>Take</th>
<th>T/P</th>
<th>T/PP</th>
<th>T/PPP</th>
<th>(\lambda \mid (\sigma^2))</th>
<th>(\lambda_{lo})</th>
<th>(\lambda_{hi})</th>
<th>(q)</th>
<th>(-L^*)</th>
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</thead>
<tbody>
<tr>
<td>n</td>
<td>T</td>
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<td>.246</td>
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<tr>
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<td>.320</td>
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<td>.320</td>
</tr>
</tbody>
</table>

What do we learn from that table?
A simple example

- We can thus plot the equilibrium path when $\lambda$ varies.

- So, once we get an observed value for $p_T$ and $p_L$ we can find the value of $\lambda$ that fits best according to some criteria of good fit.
Reading list

Presentation of the QRE model:

- Palfrey and McKelvey (1992) "An Experimental Study of the Centipede Game, Econometrica"
- MacKelvey and Palfrey (1998), Quantal response equilibria for extensive form games, Experimental Economics
- To get deeper insights into logit decision models:
- Daniel McFadden Nobels lecture, entitled Economic Choices American Economic Review 2001